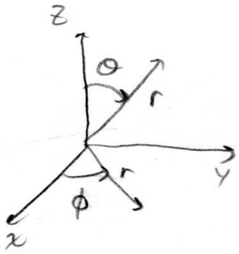
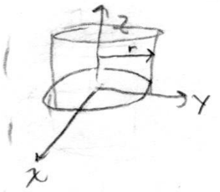
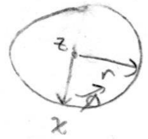


Cylindrical



Birds eye view



differential variables

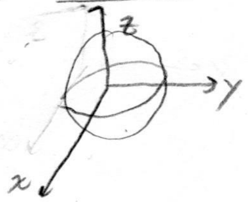
$r \longrightarrow dr$

$\phi \longrightarrow r\,d\phi$

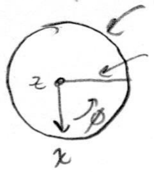
$z \longrightarrow dz$

| check units

Spherical



circle varies with height



$r\sin\theta$

$r \rightarrow dr$

$\theta \rightarrow r\,d\theta$

$\phi \rightarrow r\sin\theta\,d\phi$

Side view

- Mass of an object & density

$M = \rho V$ or σA or $\lambda \ell \Rightarrow M = \int \rho(x,y,z)\,dV$

Ex: Integrate density over surface of sphere: $\sigma(\theta,\phi) \Rightarrow \int_0^{2\pi}\int_0^{\pi} \sigma(\theta,\phi)\, r^2 \sin\theta\, d\theta\, d\phi$
(r = constant)

- Dot product & cross product
Vectors perpendicular? angle between vectors? Recall $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$
$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta \Rightarrow$ Trick

ex: $(\vec{A}\times\vec{B})_x = a_y b_z - a_z b_y$
↑ cw − ↑ ccw

- Making Approximations

taylor expansion: $f(x) = \sum_{i=0}^{\infty} \frac{d^i}{dx^i} f(x)\Big|_{x=0} \frac{x^i}{i!} \Rightarrow$

$\sin x = x - \frac{x^3}{3!} + \dots$

$\cos x = 1 - \frac{x^2}{2!} + \dots$

$\ln(1+x) = x - \frac{x^2}{2!} + \dots$

$e^x = 1 + x + \frac{x^2}{2!} + \dots$

$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \dots$

if $x \ll 1 \Rightarrow$ small angle $\Rightarrow \sin x \simeq x \quad \cos x \simeq 1$

binomial $\Rightarrow (1+x)^n \simeq 1 + nx$

$\ln(1+x) \simeq x$

ex: $\left(\frac{1}{1-x}\right)^2 = (1-x)^{-2} \simeq 1 + 2x$

Common derivatives and integrals

Power rule ex: $y = \frac{1}{x^2} + \frac{1}{x} + x^3 \Rightarrow y' = -\frac{2}{x^3} - \frac{1}{x^2} + 3x^2$

exponentials ex: $y = e^{5x}$ $\quad y' = 5e^{5x}$

Product rule ex: $y = xe^{-x^2}$ $\quad y' = e^{-x^2} + x(-2x)e^{-x^2}$

Integrals: $\int_a^b y(x)\,dx$ $\quad y = \frac{1}{x} \Rightarrow \ln\left(\frac{b}{a}\right) = \ln(b) - \ln(a)$

$= \frac{1}{x^2} \Rightarrow -\frac{1}{x}\Big|_a^b = \frac{1}{a} - \frac{1}{b}$

$= e^{-x} \Rightarrow -e^{-x}\Big|_a^b = e^{-a} - e^{-b}$

special

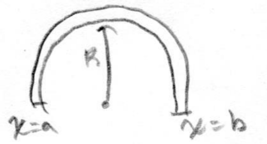
Calculate mass

thin bar 

with density $= \lambda \left[\frac{kg}{m}\right]$ $\quad M = \int \lambda(x)\,dx \;[kg]$

Ex $\lambda(x) = \alpha$ $\quad M = \int_a^b \alpha\,dx = \alpha(b-a) = \alpha l$

$\lambda(x) = \alpha x^2$ $\quad M = \int_a^b \alpha x^2\,dx = \alpha\int_a^b x^2\,dx = \frac{\alpha}{3}(b^3 - a^3)$

Circular bar 

still linear
[use cylindrical w/ R and θ constant]

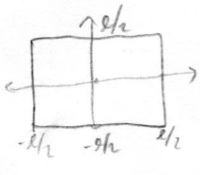
$M = \int \lambda(\phi)\,R\,d\phi$! [kg] check units

$\lambda = \alpha$ $\quad M = \int_0^\pi \alpha R\,d\phi = \alpha\pi R$ check units $\Rightarrow \left[\frac{kg}{m}\right][m]$ ✓

Check if $R = \frac{l}{\pi}$ or $\pi R = l \Rightarrow \alpha l$ ✓

$\lambda = \alpha\sin\phi$ $\quad M = \int_0^\pi \alpha\sin\phi\,R\,d\phi = \alpha R\int_0^\pi \sin\phi\,d\phi = \alpha R(-\cos\phi)\Big|_0^\pi$

$= \alpha R[1+1] = 2\alpha R$

Square plate 

with density $\sigma = \left[\frac{kg}{m^2}\right]$ $\quad M = \int \sigma(x,y)\,dA$

$\sigma(x,y) = \alpha$ $\quad M = \int_{-l/2}^{l/2}\int_{-l/2}^{l/2} \alpha\,dx\,dy = \alpha\left[\frac{l}{2} + \frac{l}{2}\right]^2 = \alpha l^2$

Sphere  with density $\rho(\theta,\phi,r)\,[\frac{kg}{m^3}]$. $M = \int_0^R \int_0^{2\pi} \int_0^{\pi} \rho(r,\theta,\phi)\, r^2 \sin\theta \, d\theta \, d\phi \, dr$

- Surface Area:

Cylindrical shell: $\int_0^{\ell} dz \int_0^{2\pi} r \, d\phi$

Spherical shell: $\int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi = 4\pi r^2$ ✓
(r is const)

- Differential Equation $\frac{dq}{dt} = -\delta q$ decay exponential

$\frac{d^2 q}{dt^2} = -k^2 q \Rightarrow$ complex exponential
lin. of sin, cosine.

Solutions $q(t) = e^{-\delta t}$

$q(t) = A\cos kt$; $B\sin kt$; $A\sin kt + B\cos kt$; Ce^{-ikt}