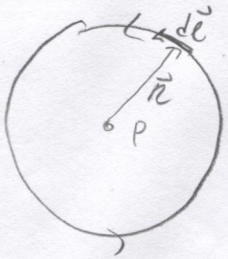
Comments on Ampere's law / Biot-Savart law

Magnetic field of wire loop



What is $\vec{B}$ at P (the center of the loop?)

Use Biot-Savart law! Ampere's law is a little tricky. But, I will do both,

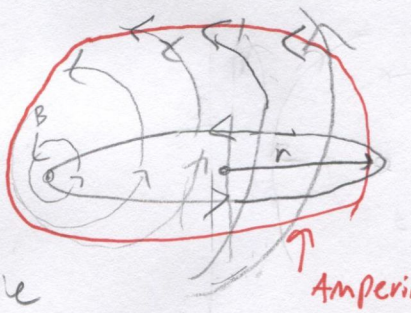
Biot-Savart

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\ell}\times\vec{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{r\,d\phi}{r^2} = \frac{\mu_0 I}{4\pi r}(2\pi) = \frac{\mu_0 I}{2r}\odot$$ (out of page)

Ampere's law

The loop must enclose the current and go <u>through</u> the center. You also want the loop to go thru the <u>entire central region</u> which means you will get...

Now, think about $\vec{B}_{wire}$... (due only to side enclosed)

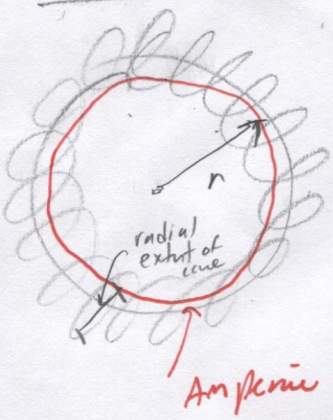


I can imagine (a better picture) where my loop is aligned with $\vec{B}$ in this way. Since, the only contribution to $\oint \vec{B}\cdot d\vec{\ell}$ is the side aligned with $\vec{B}$ (the bottom end and top end can't <u>both</u> be aligned...

$$\oint \vec{B}\cdot d\vec{\ell} = B\oint d\ell = B\,2r \Rightarrow \boxed{B = \frac{\mu_0 I}{2r}}$$

$2r$ = length between ends of loop (diameter)

Magnetic field of toroidal solenoid



I can easily use Ampère's law. Here $I_{enc} = NI$ ... so

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 NI \Rightarrow \vec{B} = \frac{\mu_0 NI}{2\pi r}\hat{r}$$

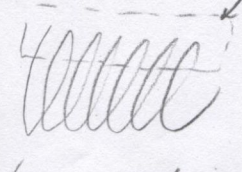
Note that $\vec{B}$ is NOT UNIFORM throughout the center. Hmm, if radial extent of the core is small compared to r, the $\hat{r}$ the variation is slight, and it is essentially uniform.

In this case, $B_{\text{toroidal solenoid}} \approx B_{\text{long solenoid}}$

Note! I use "N" to denote the total number of coils. Be very careful with this. In your text, you have "n" which represents the # of loops per length! So, for a regular solenoid, $B = \frac{\mu_0 NI}{\ell}$ becomes $\mu_0 nI$.

So, in this case, you may estimate (if the problem calls for it) $B = \frac{\mu_0 NI}{2\pi r} \approx \mu_0 nI$.

Recall: - For $B_{\text{long solenoid}}$, take amperian loop along the length of the core.



So, $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$

$B\ell = N\mu_0 I$ but if $n = \frac{N}{\ell}$

$B\ell = n\ell\mu_0 I$

$\boxed{B = n\mu_0 I}$ As stated.