SECTION: NAME:

The Spherical Capacitor: Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge +Q and outer radius  $r_a$ , and the outer shell has charge -Q and inner radius  $r_b$ .

(10 pts) **Problem 1:** Find the capacitance for the spherical capacitor.

## Electric Field

(1 pt)(i) By considering a spherical Gaussian surface, you can obtain the electric field for the region between the two spherical shells. What is  $Q_{enc}$  for the constructed surface?

(3 pts)(ii) Integrate  $\oint \vec{E} \cdot d\vec{a}$ , which runs over the entire Gaussian surface. Note:  $\vec{E}$  is uniform,  $\oint da$  is the total area of the Gaussian surface when  $\vec{E}$  is parallel to the surface normal  $\hat{n}$ .

(1 pt)(iii) What is  $\vec{E}$  (with direction  $\hat{r}$ )?

## Potential Difference

(3 pts)(i) Using  $V_{ab} = \int_{r_a}^{r_b} E(r) dr$ , find the potential difference between the concentric shells by integrating the electric field previously found.

## Capacitance

(1 pt)(i) What is the equation for the capacitance in terms of the potential difference  $V_{ab}$  and charge Q?

(1 pt)(ii) Using the value of  $V_{ab}$  you obtained, write the capacitance of the spherical capacitor.

(5 pts) **Problem 2:** Find the energy stored in the spherical capacitor.

The work dW needed to put a charge of dq on the capacitor is given by dW = Vdq.

(1 pt)(i) Write the potential V in terms of Q and C.

(2 pts)(ii) The total work is given by  $W = \int_0^W dW$ . Integrate this expression to find W.

(1 pt)(iii) What is the electrical potential energy U stored for the spherical capacitor?

(1 pt)(iv) Using the value obtained for the capacitance in problem 1, write the potential energy stored for the spherical capacitor.

## $[BONUS\ (2\ pts)]$ Conceptual Question : Circle the correct choices to complete the statement.

The capacitance of a spherical capacitor INCREASES/DECREASES with increasing radial difference  $r_b - r_a$  between the two shells, and DOUBLES/HALVES if  $r_a$  is doubled.