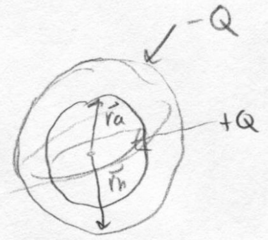
Quiz 4A

PHYSICS 208, WINTER 2016

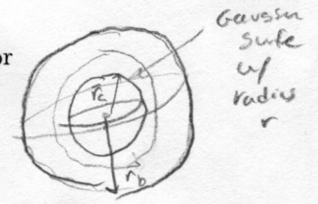
SECTION: NAME:



The Spherical Capacitor: Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge $+Q$ and outer radius r_a, and the outer shell has charge $-Q$ and inner radius r_b.

(10 pts) **Problem 1:** Find the capacitance for the spherical capacitor.

Electric Field



(1 pt)(i) By considering a spherical Gaussian surface, you can obtain the electric field for the region between the two spherical shells. What is Q_{enc} for the constructed surface?

$\boxed{Q_{enc} = Q}$

(3 pts)(ii) Integrate $\oint \vec{E} \cdot d\vec{a}$, which runs over the entire Gaussian surface.
Note: $\vec{E}$ is uniform, $\oint da$ is the total area of the Gaussian surface when $\vec{E}$ is parallel to the surface normal $\hat{n}$.

$$\oint \vec{E} \cdot d\vec{a} = E \underbrace{\oint d\vec{a}}_{\text{over the gaussn sphe}} = E A_{sphe} = E(4\pi r^2)$$

(1 pt)(iii) What is $\vec{E}$ (with direction $\hat{r}$)?

$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}}$$

Potential Difference

(3 pts)(i) Using $V_{ab} = \int_{r_a}^{r_b} E(r)\, dr$, find the potential difference between the concentric shells by integrating the electric field previously found.

$$V_{ab} = \int_{r_a}^{r_b} \frac{Q}{4\pi\epsilon_0 r^2}\, dr = \frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_a}^{r_b} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$$\boxed{V_{ab} = \frac{Q}{4\pi\epsilon_0}\left[\frac{r_b - r_a}{r_a r_b}\right]}$$

Capacitance

(1 pt)(i) What is the equation for the capacitance in terms of the potential difference V_{ab} and charge Q?

(1 pt)(ii) Using the value of V_{ab} you obtained, write the capacitance of the spherical capacitor.

$$C = \frac{Q}{V_{ab}} \Rightarrow C = \frac{\cancel{Q}}{\left(\frac{\cancel{Q}}{4\pi\epsilon_0}\right)\left(\frac{r_b - r_a}{r_a r_b}\right)} = \frac{4\pi\epsilon_0 (r_a r_b)}{r_b - r_a}$$

(5 pts) **Problem 2:** Find the energy stored in the spherical capacitor.

The work dW needed to put a charge of dq on the capacitor is given by $dW = Vdq$.

(1 pt)(i) Write the potential V in terms of Q and C. $V_{ab} = \frac{Q}{C}$

(2 pts)(ii) The total work is given by $W = \int_0^W dW$. Integrate this expression to find W.

$W = \int_0^W dw = \int_0^Q V dq = \int_0^Q \frac{Q}{C} dq = \frac{Q^2}{2C}$

(1 pt)(iii) What is the electrical potential energy U stored for the spherical capacitor?

$W = U \Rightarrow U = \frac{Q^2}{2C}$

(1 pt)(iv) Using the value obtained for the capacitance in problem 1, write the potential energy stored for the spherical capacitor.

$U = \frac{Q^2}{2C} \Rightarrow U = \frac{Q^2}{2\left(\frac{4\pi\epsilon_0 (r_a r_b)}{r_b - r_a}\right)} = \frac{Q^2 (r_b - r_a)}{8\pi\epsilon_0 (r_a r_b)}$

[BONUS (2 pts)]**Conceptual Question : Circle the correct choices to complete the statement.**
The capacitance of a spherical capacitor *INCREASES*/*DECREASES* with increasing radial difference $r_b - r_a$ between the two shells, and *DOUBLES*/*HALVES* if r_a is doubled.

$C = 4\pi\epsilon_0 \left(\frac{r_a r_b}{r_b - r_a}\right)$

Quiz 4B

PHYSICS 208, WINTER 2016

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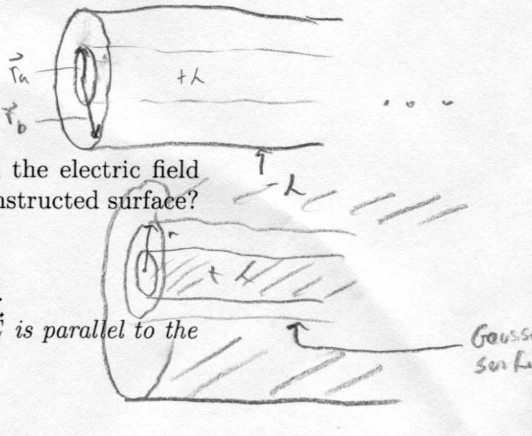
The Cylindrical Capacitor: Two long, coaxial cylindrical conductors are separated by vacuum. The inner cylinder has radius r_a and linear charge density $+\lambda$. The outer cylinder has inner radius r_b and linear charge density $-\lambda$.



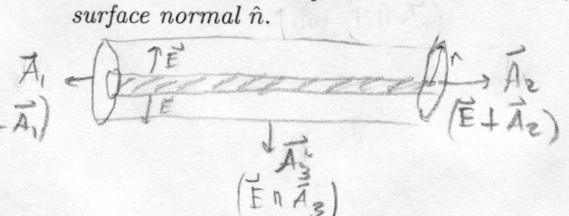
(10 pts) **Problem 1:** Find the capacitance for the cylindrical capacitor.

Electric Field

(1 pt) (i) By considering a cylindrical Gaussian surface, you can obtain the electric field for the region between the two cylindrical shells. What is Q_{enc} for the constructed surface?

$Q_{enc} = \lambda \ell$

(3 pts) (ii) Integrate $\oint \vec{E} \cdot d\vec{a}$, which runs over the entire Gaussian surface.
Note: $\vec{E}$ is uniform, $\oint da$ is the total area of the Gaussian surface when $\vec{E}$ is parallel to the surface normal $\hat{n}$.



$\oint \vec{E} \cdot d\vec{a} = E \oint d\vec{a}$ on all sides

$= E \int dA_3 = E(2\pi r \ell)$

(1 pt) (iii) What is $\vec{E}$ (with direction $\hat{r}$)?

so, $E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\lambda \ell}{(2\pi r \ell)\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}\hat{r}}$

Potential Difference

(3 pts)(i) Using $V_{ab} = \int_{r_a}^{r_b} E(r)\, dr$, find the potential difference between the concentric cylinders by integrating the electric field previously found.

$V_{ab} = \int_{r_a}^{r_b} \left(\frac{\lambda}{2\pi\epsilon_0}\right) \frac{1}{r} = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)$ $\quad \boxed{V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)}$

Capacitance

(1 pt)(i) What is the equation for the capacitance in terms of the potential difference V_{ab} and charge Q?

(1 pt)(ii) Using the value of V_{ab} you obtained, write the capacitance of the cylindrical capacitor.

$C = \frac{Q}{V_{ab}} \Rightarrow C = \frac{Q}{\left(\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)\right)} = \frac{2\pi\epsilon_0 Q}{\lambda \ln\left(\frac{r_b}{r_a}\right)}$

or $\Rightarrow \boxed{\frac{C}{\ell} = \frac{2\pi\epsilon_0}{\ln\left(\frac{r_b}{r_a}\right)}}$

(per unit length)

(5 pts) **Problem 2:** Find the energy stored in the cylindrical capacitor.

The work dW needed to put a charge of dq on the capacitor is given by $dW = Vdq$.

(1 pt)(i) Write the potential V in terms of Q and C. $\boxed{V_b = \frac{Q}{C}}$

(2 pt)(ii) The total work is given by $W = \int_0^W dW$. Integrate this expression to find W.

$dW = Vdq = \frac{Q}{C}dq \Rightarrow W = \int_0^W dW = \int_0^Q \frac{Q}{C}dq = \frac{Q^2}{2C}$

(1pt)(iii) What is the electrical potential energy U stored for the cylindrical capacitor?

$U = \frac{Q^2}{2C}$ $(W = U)$

(1pt)(iv) Using the value obtained for the capacitance in problem 1, write the potential energy stored for the cylindrical capacitor.

Here $U = \frac{Q^2}{2C} = \frac{Q^2}{2\left(\frac{2\pi\epsilon_0 Q}{\lambda \ln(\frac{r_b}{r_a})}\right)} = \frac{\lambda \ln(\frac{r_b}{r_a}) Q}{4\pi\epsilon_0} = \frac{\lambda^2 \ell \ln(\frac{r_b}{r_a})}{4\pi\epsilon_0}$ or

$\frac{U}{\ell} = \frac{\lambda \ell \ln(\frac{r_b}{r_a})}{4\pi\epsilon_0}$

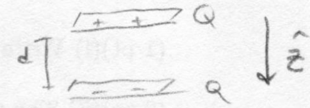
$\boxed{\frac{U}{\ell} = \frac{Q \ln(\frac{r_b}{r_a})}{4\pi\epsilon_0}}$

[BONUS (2 pts)]**Conceptual Question : Circle the correct choices to complete the statement.**
The capacitance of a cylindrical capacitor *INCREASES/DECREASES* if a dielectric material is inserted into the space between the cylinders and *INCREASES/DECREASES* if the ratio $\frac{r_b}{r_a}$ increases.

$C = \frac{2\pi\epsilon_0 Q}{\lambda \ln(\frac{r_b}{r_a})}$

Quiz 4C PHYSICS 208, WINTER 2016

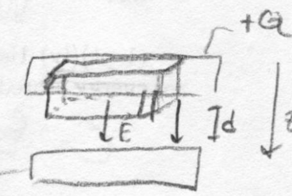
SECTION: NAME:



The Parallel-Plate Capacitor: Consider a parallel-plate capacitor with plate separation d and charge $+Q$ and $-Q$ on the top and bottom plates respectively. The positive $\hat{z}$ direction is pointing downwards.

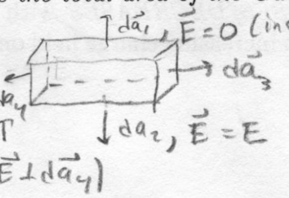
(10 pts) **Problem 1:** Find the capacitance for the parallel-plate capacitor.

Electric Field



(1 pt)(i) By considering a rectangular Gaussian surface (with one end embedded inside of a plate), you can obtain the electric field for the region between the two plates. What is Q_{enc} for the constructed surface? $\boxed{Q_{enc} = Q}$

(3 pts)(ii) Integrate $\oint \vec{E} \cdot d\vec{a}$, which runs over the entire Gaussian surface.
Note: $\vec{E}$ is uniform, $\oint da$ is the total area of the Gaussian surface when $\vec{E}$ is parallel to the surface normal $\hat{n}$.



$\oint \vec{E} \cdot d\vec{a} = E \oint d\vec{a}$ over all sides $\Rightarrow E \oint d\vec{a} = E \int da_2 = E A_{plate}$

(1 pt)(iii) What is $\vec{E}$ (with direction $\hat{z}$)?

So, $E A_{plate} = \frac{Q}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{\epsilon_0 A} \hat{z}}$

Potential Difference

(3 pts)(i) Using $\Delta V = \int_0^d E(z)\, dz$, find the potential difference between the parallel plates by integrating the electric field previously found.

$$\Delta V = \int_0^d \frac{Q}{\epsilon_0 A}\, dz = \frac{Qd}{\epsilon_0 A}$$

Capacitance

(1 pt)(i) What is the equation for the capacitance in terms of the potential difference ΔV and charge Q?

(1 pt)(ii) Using the value of ΔV you obtained, write the capacitance of the parallel-plate capacitor.

$$C = \frac{Q}{\Delta V} \Rightarrow C = \frac{\cancel{Q}}{\left(\frac{\cancel{Q}d}{\epsilon_0 A}\right)} = \frac{\epsilon_0 A}{d}$$

(5 pts) **Problem 2:** Find the energy stored in the parallel-plate capacitor.

The work dW needed to put a charge of dq on the capacitor is given by $dW = Vdq$.

(1 pt)(i) Write the potential $\cancel{\Delta V}$ V in terms of Q and C. $C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C}$

(2 pt)(ii) The total work is given by $W = \int_0^W dW$. Integrate this expression to find W.

$dW = V dq = \frac{Q}{C} dq \Rightarrow W = \int_0^W dW = \int_0^Q \frac{Q}{C} dq \Rightarrow \boxed{W = \frac{Q^2}{2C}}$

(1 pt)(iii) What is the electrical potential energy U stored for the parallel-plate capacitor?

$W = U \Rightarrow \boxed{U = \frac{Q^2}{2C}}$

(1 pt)(iv) Using the value obtained for the capacitance in problem 1, write the potential energy stored for the parallel-plate capacitor.

$C = \frac{\epsilon_0 A}{d} \Rightarrow \boxed{U = \frac{Q^2}{2\left(\frac{\epsilon_0 A}{d}\right)} = \frac{Q^2 d}{2\epsilon_0 A}}$

[BONUS (2 pts)]**Conceptual Question : Circle the correct choices to complete the statement.**
The capacitance of a parallel-plate capacitor *INCREASES/DECREASES* with increasing plate separation d and *INCREASES/DECREASES* with increasing charge held on the plates (by the capacitor).